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Mass loss of stars in star clusters: an energy source for dynamical evolution

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Abstract. Dense star clusters expand until their sizes are limited by the tidal field of their host galaxy. During this expansion phase the member stars evolve and lose mass. We show that for clusters with short initial relaxation time scales ($\lesssim$100 Myr) the dynamical expansion is largely powered by mass loss from stars in the core, but happens on a relaxation time scale. That is, the energy release following stellar mass loss is in balance with the amount of energy that is transported outward by two-body relaxation.

1. Introduction

Like a star, a stellar cluster produces heat by contracting. The positive energy that is released by the more tightly bound core is diffused outward by two-body relaxation (Lynden-Bell & Eggleton 1980) on a relaxation time scale (equation 1, Spitzer & Hart 1971). Just as for a star, the contraction phase in a cluster is only a small fraction of the total lifetime[1]. Hénon (1975) had the insight that the evolution of clusters after core contraction can be understood by assuming that an energy source provides the right amount of energy in a self-regulating fashion to sustain the two-body relaxation process. A breakthrough in the understanding of cluster evolution, comparable to Eddington's work on stellar structure.

In single-mass clusters the energy is supplied by binary stars (Hénon 1961) that form in the densest phase of core collapse (Tanikawa et al. 2012). They provide energy by interacting with other stars, thereby increasing the total (negative) energy of the cluster (Heggie 1975). In more realistic clusters mass loss of member stars also increases the total (negative) energy. In this contribution we show that the corresponding energy production rate can come into balance with the flow of energy due to two-body relaxation.

2. Time scales

When considering mass loss of stars in a collisional system there are a few time scales that need to be considered: the timescale for massive stars to segregate to the cluster core by dynamical friction, and the lifetime of stars, or main sequence time, $\tau_{\rm ms}$. The former depends on the mass of the star, m, the mean stellar mass, $\langle m\rangle$ and the half-mass

[1]This is true for clusters that are initially much denser than the density within the tidal radius.

relaxation time scale of the cluster, $\tau_{\rm rh}$, as $\tau_{\rm df} \simeq (m/\langle m\rangle)^{-1}\tau_{\rm rh}$ (Spitzer 1969). We use the conventional expression for $\tau_{\rm rh}$ (Spitzer & Hart 1971)

$$\tau_{\rm rh} = 0.138\frac{N^{1/2}r_{\rm h}^{3/2}}{\sqrt{G\langle m\rangle}\ln\Lambda}. \tag{1}$$

Here N is the number of stars, $r_{\rm h}$ is the half-mass radius, G is the gravitational constant and $\ln\Lambda$ is the Coulomb logarithm. The argument of the Coulomb logarithm is $\Lambda \simeq 0.11N$ for single-mass systems (Giersz & Heggie 1994) and $\Lambda \simeq 0.02N$ for clusters with a wide mass spectrum (Giersz & Heggie 1996). The mass-loss rate is predominantly set by $\tau_{\rm ms}$ and this timescale depends on the properties of the star, mainly m, and not on the dynamical state of the cluster. The two timescales $\tau_{\rm df}$ and $\tau_{\rm ms}$ become comparable for the most massive main sequence star after some period of time. This is because $\tau_{\rm ms} \propto m^{-\lambda}$, with $2.5 \lesssim \lambda \lesssim 3$, and $\tau_{\rm df} \propto m^{-1}$. A star for which $\tau_{\rm df}(m) = \tau_{\rm ms}(m)$ reaches the core within its lifetime and will lose its mass there. From that moment onwards mass loss will proceed on average from the central parts of the cluster. The amount of energy that is released depends on the mass and specific energy of the star at the moment it loses mass. In § 3 we quantify how much energy a cluster needs to evolve and in § 4 we show how much energy can be generated by mass loss.

3. Energy demand

Let us first consider how much energy is needed per unit of time. Just as for a star, the flow of energy, or luminosity $\dot{E}$, is determined by the properties of the system as a whole. It equals a fraction ζ of the instantaneous total energy E per unit of $\tau_{\rm rh}$ (Hénon 1961; Heggie & Hut 2003; Gieles et al. 2011)

$$\frac{\dot{E}}{E} = -\frac{\zeta}{\tau_{\rm rh}}. \tag{2}$$

The total energy for a cluster in virial equilibrium may be written as

$$E = -\kappa\frac{GM^2}{r_{\rm h}}, \tag{3}$$

where $\kappa \simeq 0.20 - 0.25$, depending on the density profile and $M = \langle m\rangle N$ is the total mass. The evolution of the main cluster parameters (N and $r_{\rm h}$) can be understood once we know how $\tau_{\rm rh}$ evolves. For this it is insightful to consider the idealised models of Hénon. In the absence of a tidal field the energy flow leads to a self-similar expansion at a roughly constant mass and $\tau_{\rm rh}$ grows linearly in time such that $r_{\rm h} \propto t^{2/3}$ (Hénon 1965). In a steady tidal field the radius is limited by the tidal radius and $\tau_{\rm rh}$ decreases linearly in time (Hénon 1961). In both cases $\zeta \simeq 0.1$ (see the derivation from Hénon's work in Gieles et al. 2011 and see Alexander & Gieles 2012 for measurements of ζ from numerical simulations) and ζ can be more than an order of magnitude higher for clusters with a wide mass spectrum (Gieles et al. 2010).

We consider clusters that are not strongly confined by a tidal field. This is justified by the observation that about half of the (old) Galactic globular clusters are still expanding towards their tidal boundary (Baumgardt et al. 2010; Gieles et al. 2011). We then find from equation (1) and (2) that $\tau_{\rm rh} = (3/2)\zeta t$ and

$$\frac{\dot{E}}{E} = -\frac{2/3}{t}. \tag{4}$$

This means that the (absolute value of the) cluster energy decreases by roughly 80% each age dex during the expansion. It is in this phase that stellar mass loss can provide the energy. Although a cluster only loses about half of its initial mass due to stellar mass loss over a Hubble time, this modest mass-loss rate in fact generates energy at a rate comparable to what is required (equation 4), as we will show in § 4.

4. Energy supplied by mass loss

As we aim to show in this section, the rate of energy increase as a result of mass loss of stars, $\dot{E}_\Delta$, leads to a power-law decline of $-E$ (equation 4). This is primarily because the evolution of the total mass as a result of stellar evolution from a stellar system with a Salpeter like stellar initial mass function, declines roughly as a power of time $M(t) \propto t^{-\nu}$, with $\nu \simeq 0.07$ (this corresponds to a loss of about 15% of the mass every age dex, e.g. Bruzual & Charlot 2003). The instantaneous mass-loss rate is thus

$$\frac{\dot{M}}{M} = -\frac{\nu}{t}. \tag{5}$$

To relate this to energy we need to know from where and how fast mass is lost. We assume that the mass-loss time scale is much longer than the local crossing time, such that the cluster responds adiabatically and retains virial equilibrium. Hills (1980) discussed the adiabatic response of a gravitational system if mass is lost homologous with the density profile of the cluster, that is, without a preferred location. The cluster radius then increases as M^{-1} and the change in energy relates to the change in mass as (equation 3)

$$\frac{\mathrm{d}E}{E} = 3\frac{\mathrm{d}M}{M}. \tag{6}$$

When $\tau_{\mathrm{df}} << \tau_{\mathrm{ms}}$ mass loss happens predominantly from the core of the cluster, because the most massive stars, i.e. the ones contributing most to the total stellar mass loss, are centrally concentrated because of dynamical friction[2]. Mass loss affects both the potential energy $U = 2E$ and the kinetic energy $T = (1/2)M\langle v^2\rangle$, where $\langle v^2\rangle$ is the mean-square velocity of the system. This is because the mass that is removed carries away a bit of potential and kinetic energy and because of the subsequent redistribution of potential and kinetic energy of the remaining stars. We can express the change in the potential energy in terms of the specific potential of the mass that is lost, ϕ_Δ, relative to the average specific potential of all the stars, $\langle\phi\rangle = 2U/M$, as $\mathrm{d}U/U = 2(\phi_\Delta/\langle\phi\rangle)\mathrm{d}M/M$. For the kinetic term we need to compare the squared velocity of the mass that is lost, v_Δ^2, to the mean square velocity of the cluster: $\mathrm{d}T/T = (v_\Delta^2/\langle v^2\rangle)\mathrm{d}M/M$. Using our assumption that the remaining stars quickly restore virial equilibrium ($E = U/2 = -T$) we can relate the change in the total energy to the change in mass

[2]This simple view can get slightly more complicated if the turn-off mass is significantly lower than the non-evolving remnants. If many remnants are retained they will eventually dominate the dynamics in the inner regions of cluster.

$$
\begin{aligned}
\frac{\mathrm{d}E}{E} &= 2\frac{\mathrm{d}U}{U} - \frac{\mathrm{d}T}{T}, &(7)\\
&= x\frac{\mathrm{d}M}{M}, \qquad \text{with } x \equiv 4\frac{\phi_\Delta}{\langle\phi\rangle} - \frac{v_\Delta^2}{\langle v^2\rangle}. &(8)
\end{aligned}
$$

For homologous mass loss $\langle\phi_\Delta\rangle = \langle\phi\rangle$ and $\langle v_\Delta^2\rangle = \langle v^2\rangle$ and therefore $x = 3$. This is Hills' result (equation 6). The general result for the rate of energy 'supply' is

$$\frac{\dot{E}_\Delta}{E} = -\frac{x\nu}{t}. \tag{9}$$

From a comparison to the energy 'demand' (equation 4) we see that *a balance between energy supply and demand is reached if* $x\nu \simeq 2/3$*.* Because $\nu \simeq 0.07$ the condition for energy generation by stellar mass loss to be in balance by two-body relaxation becomes $x \simeq 10$. To answer the question whether this value is realistic, we need to consider a cluster potential that described a cluster in the balanced evolution phase. One example is the Jaffe (1983) model, which has the required r^{-2} density cusp. From the potential of this model we find $\phi(r)/\langle\phi\rangle = \ln(1 + r_\mathrm{h}/r)$. If we ignore the small contribution of the kinetic term to x in equation (8) we find that $x \simeq 10$ if mass loss takes place at $r \simeq 0.1r_\mathrm{h}$. This is probably realistic in the case of a mass segregated system. This shows that the product $x\nu$ can be of sufficient magnitude (about $2/3$, equation 4) to drive the relaxation process. In a forthcoming paper (Gieles, Heggie & Church) we will illustrate this result with numerical simulations and show that stellar mass loss is indeed a viable energy source in driving the dynamical expansion of star clusters.

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References

Alexander, P., & Gieles, M. 2012, MNRAS, 422, 3415
Baumgardt, H., Parmentier, G., Gieles, M., & Vesperini, E. 2010, MNRAS, 401, 1832.
Bruzual, G., & Charlot, S. 2003, MNRAS, 344, 1000
Gieles, M., Baumgardt, H., Heggie, D. C., & Lamers, H. J. G. L. M. 2010, MNRAS, 408, L16.
Gieles, M., Heggie, D. C., & Zhao, H. 2011, MNRAS, 413, 2509.
Giersz, M., & Heggie, D. C. 1994, MNRAS, 268, 257.
— 1996, MNRAS, 279, 1037.
Heggie, D., & Hut, P. 2003, The Gravitational Million-Body Problem: A Multidisciplinary Approach to Star Cluster Dynamics (Cambridge University Press, 2003, 372 pp.)
Heggie, D. C. 1975, MNRAS, 173, 729
Hénon, M. 1961, Annales d'Astrophysique, 24, 369; English transl: arXiv:1103.3499
— 1965, Annales d'Astrophysique, 28, 62; English transl: arXiv:1103.3498
— 1975, in Proc. IAU Symp. 69, Dynamics of the Solar Systems. Reidel, Dordrecht, p. 133
Hills, J. G. 1980, ApJ, 235, 986
Jaffe, W. 1983, MNRAS, 202, 995
Jeans, J. H. 1961, Astronomy and cosmogony (New York: Dover)
Lynden-Bell, D., & Eggleton, P. P. 1980, MNRAS, 191, 483
Spitzer, L. J. 1969, ApJ, 158, L139
Spitzer, L. J., & Hart, M. H. 1971, ApJ, 164, 399
Tanikawa, A., Hut, P., & Makino, J. 2012, New A, 17, 272.